

ICRANet



UNIVERSIDAD  
DE ANTIOQUIA

1803

Addendum to the COOPERATION  
AGREEMENT 21401601-233

BETWEEN

UNIVERSIDAD DE ANTIOQUIA

AND

ICRANET

Adenda del

ACUERDO DE COOPERACIÓN  
21401601-233

ENTRE LA

UNIVERSIDAD DE ANTIOQUIA

E

ICRANET

This is an addendum to the cooperation agreement 21401601-233 between University of Antioquia and ICRANET, according to clause 2.

ICRANET will fund the attached research project within the research line of "Cosmology and Large Scale Structures", which is one of the research lines approved by the ICRANET Scientific Committee.

#### **Human and material Resources**

ICRANET will contribute in office premises, communication, computational facilities, housing costs of the researchers during the visits to the ICRANET centers, and assigning ICRANET researchers to work on the project as required to achieve the planned objectives.

The University of Antioquia will contribute by allowing the principal investigator, Romano Antonio Enea, to devote 10 hours per week to the planned research activities for the duration of the project.

#### **Persons responsible for the planned activities**

The persons responsible for the planned activities are:

Prof. Romano Antonio Enea, University of Antioquia, Principal Investigator at UDEA;  
Prof. Jorge Rueda, ICRANET, Principal Investigator at ICRANET.

Esta es una adenda al acuerdo de cooperación 21401601-233 entre la Universidad de Antioquia e ICRANET.

ICRANET financiará el proyecto de investigación adjunto en el marco de la línea de investigación "Cosmología y estructura de larga escala" que es una de las líneas de investigación aprobadas por el comité Científico de ICRANET.

#### **Recursos humanos y materiales**

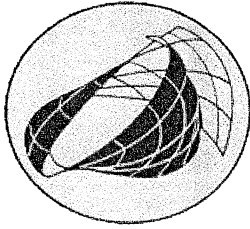
ICRANET contribuirá cubriendo los costos de oficinas, comunicaciones, equipos de cómputo, costos de alojamientos de las visitas de los investigadores a los centros ICRANET y asignando investigadores para trabajar en el proyecto como requerido para alcanzar los objetivos planeados.

La Universidad de Antioquia contribuirá con una dedicación a las actividades de investigación de 10 horas semanales del investigador principal Romano Antonio Enea por la duración del proyecto.

#### **Personas responsables de las actividades previstas**

Las personas responsables para las actividades previstas son:

Prof. Romano Antonio Enea, Universidad de Antioquia, Investigador Principal para la UDEA;  
Prof. Jorge Rueda, ICRANET, Investigador principal para ICRANET.



ICRANet

*International Center for Relativistic Astrophysics Network*

**Research Project:**

**I. TITLE**

**“Large scale inhomogeneities and the fractal structure of the Universe”**

En el marco del acuerdo de cooperación ICRANET-UDEA 21401601-233

Within to the cooperation agreement ICRANET-UDEA 21401601-233

**Principal Investigator UDEA: Prof. Romano Antonio Enea**

**Principal Investigators ICRANET: Prof. Remo Ruffini, Prof. Jorge Rueda, Prof.ssa Simonetta Filippi**

**II. LINEA DE INVESTIGACION**

El proyecto se inscribe en la línea de investigación de Cosmología y Astrofísica teórica.

**III. ANTECEDENTES**

While the standard cosmological models (SCM) is in good agreement with many cosmological and astrophysical observations, there are several unexplained inconsistencies between theory and observation which deserve attention. One of the assumptions of the SCM is that Universe can be described as a homogeneous and isotropic space time has proven to be quite successful in explaining observations, but the scale of inhomogeneity is still not clear, and there can be physical systems where the effects of the inhomogeneities could be very important, and partially explain those inconsistencies. Supernovae Ia observations have strongly supported the existence of a cosmological constant or some other form of dark energy, which should actually account for about 70% of the total energy content of the Universe. Dark energy is still an open mystery, since it has an equation of state which is different from known form of matter or radiation, and nevertheless accounts for such a large portion of the energy budget. While supernovae were initially the main evidence of the existence of dark energy, this has been confirmed independently by other observations such as the cosmic microwave background (CMB) radiation or large scale structure.

Modern cosmology has achieved an unprecedented level of accuracy in measuring cosmological observables and testing theoretical models trying to explain them. The standard cosmological model based on the assumption of large scale isotropy and homogeneity is fitting observational data quite successfully, but given the high level of accuracy reached in the estimation of its cosmological parameters it has become important to take into account what could be the effects of a local inhomogeneity. This can be done using inhomogeneous

solutions of Einstein's fields equations, but in this approach the inhomogeneities are not regarded as an alternative to dark energy as explanation of the apparent acceleration of the Universe.

The study of these effects on the estimation of cosmological parameters can be very important because they could be comparable to other uncertainties coming from systematic or statistical errors, and as such cannot be neglected.

Another pillar of modern cosmology is inflation, a period of very fast expansion without which we cannot explain the features of the observed Universe such as the flatness, or solve the so called horizon problem, which is related to the very high level isotropy of the CMB radiation. Recent measurements of the CMB polarization in the experiments BICEP2 [1] allowed to determine the strength of primordial gravitational waves, and consequently the energy scale at which inflation occurred. This has some very important implications on the determination of different cosmological parameters, and we will consider in particular how late time inhomogeneities arising from early Universe primordial curvature perturbations could affect our estimation of other cosmological parameters such as the cosmological constant.

It was also proposed that large scale structure follows a fractal distribution [11-12] and for this reason it is important to understand observations indeed support this theoretical model.

#### IV. MARCO TEORICO

In this research project we will assume that local inhomogeneities can be modeled by an inhomogeneous solution of Einstein's equations, the Lemaitre-Tolman-Bondi (LTB) metric, a spherically symmetric pressureless solution which can be written as [2-4]:

$$ds^2 = -dt^2 + (R,r)^2 / (1+2E) + R^2 d\Omega^2 \quad (1)$$

where  $R$  is a function of the time coordinate  $t$  and the radial coordinate  $r$ ,  $R = R(t,r)$ ,  $E$  is an arbitrary function of  $r$ ,  $E = E(r)$  and  $R,r = \partial R / \partial r$ . We will consider the solution in presence of a cosmological constant in order to find the effects on its value. This type of solution will be denoted as  $\Lambda$ LTB model in the rest of this document. As a first step in this investigation we will assume to be located at the center, which corresponds to take into account the monopole contribution of the local inhomogeneity.

We will then model the large scale structure making an ansatz for the mass function  $M(r)$  of the type  $a r^d$ , where  $d$  will be interpreted as the fractal dimension.

#### V. OBJETIVOS DEL PROYECTO

- Investigate the hypothesis of the fractal structure of the Universe
- Derive analytical formulae to understand the effects of inhomogeneities on the expansion rate and the luminosity distance
- Derive a consistency relation involving the luminosity distance and the expansion rate
- Determine if the observed inconsistency between the luminosity distance and the Hubble expansion rate can be explained as the effect of large scale inhomogeneity

## VI. METODOLOGIA PROPUESTA

This project will involve both a local Taylor expansion in redshift space of the relevant observables and the numerical calculation necessary to study higher redshift behavior. In particular we are interested in finding the formulae for the luminosity distance for a  $\Lambda$ LTB matter dominated solution, and in finding the effects on the estimation of the value of the cosmological constant coming from local inhomogeneities.

The fitting of the Supernovae Ia data will involve the following steps:

- Define a model for the local structure based on the ansatz  $M(r)=a r^d$
- Define the corresponding LTB model using the equation (4).
- Solve numerically the radial null geodesic equations for  $t(z),r(z)$  and the Einstein's equations for the background for  $R(t,r)$
- Calculate the theoretical luminosity distance by  $D(z) = (1 + z)2R(t(z),r(z))$  and the statistical variable,  $\chi$ , where the summation is over all the supernovae included in the data fitting.
- Run the previous steps for a two dimensional grid of values for the parameters  $\{\sigma,\Omega\Lambda\}$ , and find the minimum of  $\chi^2$  and the corresponding best fit parameters.
- Repeat the previous steps for different values of  $A = n * 5 * 10^{(-5)}$ , where we will take  $n = \{1,2,3\}$  corresponding to respectively one, two and three standard deviations of the primordial curvature perturbations as inferred from the CMB spectrum, since we are considering the effects of being located at the center of a peak of the primordial curvature perturbations.

The analytical calculation will involve the following steps:

- Find a Taylor expansion of the solutions of the geodesics equations for  $r(z)$  and  $t(z)$ .

This is achieved by Taylor expanding both the l.h.s. and the r.h.s. of the geodesics equations and then setting a system of linear algebraic equations to obtain the coefficients of the expansion of  $r(z)$  and  $t(z)$  by matching the terms of the same order in the expansion of the differential equations as shown in [9].

- Substitute the above the above expansions in the formula  $DL(z) = (1 + z)2R(t(z),r(z))$ , and expand it in powers of  $z$ .

This last step is based on the use of the analytical solution for  $R(t,r)$  given in [10] for example:

- The analytical calculations necessary to obtain a formula for the luminosity distance will be performed in MATHEMATICA due to the high complexity of the symbolical manipulation procedure necessary to find a local expansion of the solution to the geodesics equations.
- The final formula will be put in a form depending directly on the physical observable parameters such as  $H_0$ .
- The same procedure will be applied both numerically and analytically to the calculation of  $H(z)$ .

## VII. CRONOGRAMA

- meses 1-2 : Study of the literature about inhomogeneous cosmological models.
- meses 3-18 : Design and implementation of the numerical code for the fitting of observational data with LTB models.
- meses 19-36 : Writing papers and submit to scientific journals

## Referencias

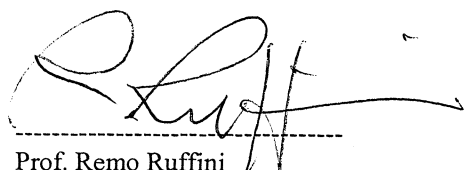
- [1] BICEP2 Collaboration, P. Ade et al., (2014), arXiv:1403.3985.
- [2] G. Lemaitre, Gen.Rel.Grav. 29, 641 (1997).
- [3] R. C. Tolman, Proc.Nat.Acad.Sci. 20, 169 (1934).
- [4] H. Bondi, Mon. Not. Roy. Astron. Soc. 107, 410 (1947).
- [5] A. E. Romano, M. Sasaki, and A. A. Starobinsky, Eur.Phys.J. C72, 2242 (2012), arXiv:1006.4735.
- [6] A. A. Starobinsky, Phys.Lett. B117, 175 (1982).
- [7] A. A. Starobinsky, JETP Lett. 42, 152 (1985).
- [8] D. H. Lyth, K. A. Malik, and M. Sasaki, JCAP 0505, 004 (2005), arXiv:astro-ph/0411220.
- [9] A. E. Romano, JCAP 1005, 020 (2010), arXiv:0912.2866.
- [10] A. E. Romano, Int.J.Mod.Phys. D21, 1250085 (2012), arXiv:1112.1777
- [11] Ruffini, R.; Song, D. J.; Taraglio, S., Astr. and Astrop. v. 190, no. 1-2, Jan. 1988, p. 1-9
- [12] Stahl, Clément, Int.J.Mod.Phys. D25 (2016) no.06, 1650066

For ICRANet:

Por UDEA:

Date 27 / Feb / 2017

Fecha \_\_\_\_\_ / \_\_\_\_\_ / \_\_\_\_\_



Prof. Remo Ruffini  
ICRANet Director



Dr. Nora Restrepo Sanchez  
Decana  
Facultad de Ciencias Exactas y Naturales  
Universidad de Antioquia